Electronic Journal of Qualitative Theory of Differential Equations
2013, No. **1**, 1-10; **http://www.math.u-szeged.hu/ejqtde/**

Periodic solutions for Nicholson-type delay system with nonlinear density-dependent mortality terms*

Zhibin Chen†

School of Science, Hunan University of Technology, Hunan Zhuzhou, 412000, P.R. China

Abstract: This paper is concerned with the periodic solutions for a class of new Nicholson-type delay system with nonlinear density-dependent mortality terms. By using coincidence degree theory, some criteria are obtained to guarantee the existence of positive periodic solutions of the model. Moreover, an example is employed to illustrate the main results.

Keywords: Nicholson-type delay system; positive periodic solution; coincidence degree; nonlinear density-dependent mortality term.
AMS(2000) Subject Classification: 34C25; 34K13

1 Introduction

In a classical study of population dynamics, the Nicholson's blowflies model

$$N'(t) = -\delta N(t) + pN(t-\tau)e^{-aN(t-\tau)}, \tag{1.1}$$

has been proposed by Gurney et al. [1] to describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained in [2]. Here $N(t)$ stands for the size the population at time t, p is the maximum per capita daily egg production, $\frac{1}{a}$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. In the past forty years, this model and its modifications have been extensively and intensively studied from both theoretical and mathematical biologists(see, for example [3-14]).

*This work was supported by the National Natural Science Foundation of China (grant no. 11201184), and the Scientific Research Fund of Hunan Provincial Natural Science Foundation of China (Grant No. 12JJ3007).
† Corresponding author. Tel.: +86073122183385, fax: +86073122183385, E-mail: chenzhibinbin@yahoo.cn

Recently, as pointed out in L. Berezansky et al. [15], a new study indicates that a linear model of density-dependent mortality will be most accurate for populations at low densities, and marine ecologists are currently in the process of constructing new fishery models with nonlinear density-dependent mortality rates. Consequently, the dynamic behaviors of the Nicholson's blowflies model with a nonlinear density-dependent mortality term have been the object of intensive analysis by numerous authors and some of these results can be found in [16-18]. In particular, B. Liu et al. [19] established the results on the permanence for the following Nicholson-type delay system with nonlinear density-dependent mortality terms:

$$\begin{cases} N_1'(t) = -(a_{11}(t) - b_{11}(t)e^{-N_1(t)}) + (a_{12}(t) - b_{12}(t)e^{-N_2(t)}) \\ \qquad\qquad + \sum\limits_{j=1}^{l} c_{1j}(t)N_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)N_1(t-\tau_{1j}(t))}, \\ N_2'(t) = -(a_{22}(t) - b_{22}(t)e^{-N_2(t)}) + (a_{21}(t) - b_{21}(t)e^{-N_1(t)}) \\ \qquad\qquad + \sum\limits_{j=1}^{l} c_{2j}(t)N_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)N_2(t-\tau_{2j}(t))}, \end{cases} \tag{1.2}$$

under the admissible initial conditions

$$x_{t_0} = \varphi, \quad \varphi \in C_+ = C([-r_1, 0], R_+^1) \times C([-r_2, 0], R_+^1) \text{ and } \varphi_i(0) > 0, \quad i = 1, 2, \tag{1.3}$$

where $a_{ij}, b_{ij}, c_{ik}, \gamma_{ik} : R^1 \to (0, +\infty)$ and $\tau_{ik} : R^1 \longrightarrow [0, +\infty)$ are all bounded continuous functions, $r_i = \max\limits_{1\le k\le l}\{\tau_{ik}^+\}$, and $i, j = 1, 2, k = 1, 2, \cdots, l$. However, to the best of our knowledge, few authors have considered the problem for positive periodic solutions of Nicholson-type delay system (1.2). On the other hand, system (1.2) can be used to describe the dynamics for the models of Marine Protected Areas and B-cell Chronic Lymphocytic Leukemia dynamics that belong to the Nicholson-type delay differential systems with nonlinear density-dependent mortality terms (see [12-14, 19]). Motivated by the above papers, in this present paper, the main purpose is to give the conditions to guarantee the existence of positive periodic solutions of system (1.2).

For convenience, we introduce some notations. Throughout this paper, given a bounded continuous function g defined on R^1, let g^+ and g^- be defined as

$$g^- = \inf_{t\in R^1} g(t), \quad g^+ = \sup_{t\in R^1} g(t).$$

We also assume that $a_{ij}, b_{ij}, c_{ik}, \gamma_{ik} : R^1 \to (0, +\infty)$ and $\tau_{ik} : R^1 \longrightarrow [0, +\infty)$ are all ω-periodic functions, and $i, j = 1, 2, k = 1, 2, \cdots, l$.

Let $R^n(R_+^n)$ the set of all (nonnegative) real vectors, $n = 1, 2$, we will use $x = (x_1, x_2)^T \in R^n$ to denote a column vector, in which the symbol $(\ )^T$ denotes the transpose of a vector. we let $|x|$ denote the absolute-value vector given by $|x| = (|x_1|, |x_2|)^T$ and define $||x|| = \max\limits_{1\le i\le 2} |x_i|$. For matrix $A = (a_{ij})_{n\times n}$, A^T denotes the transpose of A. A matrix or vector $A \geq 0$ means that all entries of A are greater than or equal to zero. $A > 0$ can be defined similarly. For matrices or vectors A and B, $A \geq B$ (resp. $A > B$) means that $A - B \geq 0$ (resp. $A - B > 0$). We also define the derivative and integral of vector function $x(t) = (x_1(t), x_2(t))^T$ as $x' = (x_1'(t), x_2'(t))^T$ and $\int_0^\omega x(t)dt = (\int_0^\omega x_1(t)dt, \int_0^\omega x_2(t)dt)^T$.

The remaining part of this paper is organized as follows. In the next section, some sufficient conditions for the existence of the positive periodic solutions of system (1.2) are given by using the method of coincidence degree. In Section 3, an example is given to illustrate our result obtained in the previous section.

2 Existence of Positive Periodic Solutions

In order to study the existence of positive periodic solutions, we first introduce the Continuation theorem as follows:

Lemma 2.1 (Mawhin's continuous theorem [20])**.** *Let X and Z be two Banach spaces. Suppose that $L : D(L) \subset X \to Z$ is a Fredholm operator with index zero and $N : X \to Z$ is* L *-compact on $\overline{\Omega}$, where Ω is an open subset of X. Moreover, assume that all the following conditions are satisfied:*

(1)*$Lx \neq \lambda Nx$, for all $x \in \partial\Omega \cap D(L), \lambda \in (0, 1)$;*

(2)*$Nx \notin ImL$, for all $x \in \partial\Omega \cap KerL$;*

(3) *The Brouwer degree*

$$deg\{QN, \Omega \cap KerL, 0\} \neq 0.$$

Then equation $Lx = Nx$ has at least one solution in $domL \cap \overline{\Omega}$.

We are now in a position to state our main result.

Theorem 2.1. *Let*

$$\frac{b_{11}^-}{a_{11}^+ + b_{12}^+ - a_{12}^-} > 1, \qquad a_{11}^- > a_{12}^+ + \sum_{j=1}^{l} \frac{c_{1j}^+}{\gamma_{1j}^- e}, \tag{2.1}$$

and

$$\frac{b_{22}^-}{a_{22}^+ + b_{21}^+ - a_{21}^-} > 1, \qquad a_{22}^- > a_{21}^+ + \sum_{j=1}^{l} \frac{c_{2j}^+}{\gamma_{2j}^- e}. \tag{2.2}$$

Then (1.2) has at least one positive $\omega-$periodic solution.

Proof. Let $N(t) = (N_1(t), N_2(t))^T$ and $N_i(t) = e^{x_i(t)} (i = 1, 2)$. Then (1.2) can be rewritten as

$$\begin{cases} x_1'(t) = -\frac{a_{11}(t)}{e^{x_1(t)}} + \frac{b_{11}(t)}{e^{x_1(t)}+e^{x_1(t)}} + \frac{a_{12}(t)}{e^{x_1(t)}} - \frac{b_{12}(t)}{e^{x_1(t)}+e^{x_2(t)}} \\ \qquad + \sum_{j=1}^{l} c_{1j}(t) \frac{e^{x_1(t-\tau_{1j}(t))}}{e^{x_1(t)+\gamma_{1j}(t)e^{x_1(t-\tau_{1j}(t))}}} := \Delta_1(x, t), \\ x_2'(t) = -\frac{a_{22}(t)}{e^{x_2(t)}} + \frac{b_{22}(t)}{e^{x_2(t)}+e^{x_2(t)}} + \frac{a_{21}(t)}{e^{x_2(t)}} - \frac{b_{21}(t)}{e^{x_2(t)}+e^{x_1(t)}} \\ \qquad + \sum_{j=1}^{l} c_{2j}(t) \frac{e^{x_2(t-\tau_{2j}(t))}}{e^{x_2(t)+\gamma_{2j}(t)e^{x_2(t-\tau_{2j}(t))}}} := \Delta_2(x, t), \end{cases} \tag{2.3}$$

As usual, let $X = Z = \{x = (x_1(t), x_2(t))^T \in C(R^1, R^2) : x(t + \omega) = x(t) \text{ for all } t \in R^1\}$ be Banach spaces equipped with the supremum norm $||\cdot||$. For any $x \in X$, because of periodicity, it is easy to see that $\Delta(x, \cdot) = (\Delta_1(x, \cdot), \Delta_2(x, \cdot))^T \in C(R^1, R^2)$ is ω-periodic. Let

$$L : D(L) = \{x \in X : x \in C^1(R^1, R^2)\} \ni x \longmapsto x' = (x_1', x_2')^T \in Z,$$

$$P : X \ni x \longmapsto \Big(\frac{1}{\omega}\int_0^\omega x_1(s)ds, \frac{1}{\omega}\int_0^\omega x_2(s)ds\Big)^T \in X,$$

$$Q : Z \ni z \longmapsto \Big(\frac{1}{\omega}\int_0^\omega z_1(s)ds, \frac{1}{\omega}\int_0^\omega z_2(s)ds\Big)^T \in Z,$$

$$N : X \ni x \longmapsto \Delta(x, \cdot) \in Z.$$

It is easy to see that

$$ImL = \{x | x \in Z, \int_0^\omega x(s)ds = (0, 0)^T\}, KerL = R^2, ImP = KerL \ \text{ and } \ KerQ = ImL.$$

Thus, the operator L is a Fredholm operator with index zero. Furthermore, denoting by $L_P^{-1} : ImL \to D(L) \cap KerP$ the inverse of $L|_{D(L)\cap KerP}$, we have

$$\begin{aligned} L_P^{-1}y(t) =& -\frac{1}{\omega}\int_0^\omega \int_0^t y(s)dsdt + \int_0^t y(s)ds \\ =&\Big(-\frac{1}{\omega}\int_0^\omega \int_0^t y_1(s)dsdt + \int_0^t y_1(s)ds, -\frac{1}{\omega}\int_0^\omega \int_0^t y_2(s)dsdt + \int_0^t y_2(s)ds\Big)^T. \end{aligned} \tag{2.4}$$

It follows that

$$QNx = \frac{1}{\omega}\int_0^\omega Nx(t)dt = \Big(\frac{1}{\omega}\int_0^\omega \Delta_1(x(t), t)dt, \frac{1}{\omega}\int_0^\omega \Delta_2(x(t), t)dt\Big)^T, \tag{2.5}$$

$$\begin{aligned} L_P^{-1}(I - Q)Nx =& \int_0^t Nx(s)ds - \frac{t}{\omega}\int_0^\omega Nx(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^t Nx(s)dsdt \\ &+ \frac{1}{\omega}\int_0^\omega \int_0^t QNx(s)dsdt. \end{aligned} \tag{2.6}$$

Obviously, QN and $L_P^{-1}(I-Q)N$ are continuous. It is not difficult to show that $L_P^{-1}(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega \subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Considering the operator equation $Lx = \lambda Nx, \lambda \in (0,1)$, we have

$$x'(t) = (x_1'(t), x_2'(t))^T = \lambda\Delta(x,t) = (\lambda\Delta_1(x,t), \lambda\Delta_2(x,t))^T. \tag{2.7}$$

Suppose that $x = (x_1(t), x_2(t))^T \in X$ is a solution of (2.7) for some $\lambda \in (0,1)$, then there exist $\xi_1, \xi_2, \eta_1, \eta_2 \in [0,\omega]$ such that

$$x_i(\xi_i) = \min_{t\in[0,\omega]} x_i(t),\ x_i(\eta_i) = \max_{t\in[0,\omega]} x_i(t), \quad \text{and} \quad x_i'(\xi_i) = x_i'(\eta_i) = 0, \quad i = 1,2. \tag{2.8}$$

It follows from (2.7) and (2.8) that

$$\begin{aligned} x_1'(\xi_1) =&\lambda\Bigg[-\frac{a_{11}(\xi_1)}{e^{x_1(\xi_1)}} + \frac{b_{11}(\xi_1)}{e^{x_1(\xi_1)+e^{x_1(\xi_1)}}} + \frac{a_{12}(\xi_1)}{e^{x_1(\xi_1)}} - \frac{b_{12}(\xi_1)}{e^{x_1(\xi_1)+e^{x_2(\xi_1)}}} \\ &+\sum_{j=1}^{l} c_{1j}(\xi_1)\frac{e^{x_1(\xi_1-\tau_{1j}(\xi_1))}}{e^{x_1(\xi_1)+\gamma_{1j}(\xi_1)e^{x_1(\xi_1-\tau_{1j}(\xi_1))}}}\Bigg] \\ =&0. \end{aligned} \tag{2.9}$$

Thus,

$$\begin{aligned} a_{11}^+ + b_{12}^+ - a_{12}^- \geq& a_{11}(\xi_1) + b_{12}(\xi_1) - a_{12}(\xi_1) \\ \geq& a_{11}(\xi_1) + \frac{b_{12}(\xi_1)}{e^{e^{x_2(\xi_1)}}} - a_{12}(\xi_1) \\ =&\frac{b_{11}(\xi_1)}{e^{e^{x_1(\xi_1)}}} + \sum_{j=1}^{l} c_{1j}(\xi_1)\frac{e^{x_1(\xi_1-\tau_{1j}(\xi_1))}}{e^{\gamma_{1j}(\xi_1)e^{x_1(\xi_1-\tau_{1j}(\xi_1))}}} \\ \geq&\frac{b_{11}(\xi_1)}{e^{e^{x_1(\xi_1)}}} \\ \geq&\frac{b_{11}^-}{e^{e^{x_1(\xi_1)}}}, \end{aligned} \tag{2.10}$$

which, together with (2.1), implies that

$$x_1(\xi_1) \geq \ln\left(\ln\frac{b_{11}^-}{a_{11}^+ + b_{12}^+ - a_{12}^-}\right) := H_{11}. \tag{2.11}$$

Combining (2.7) with (2.8), we also have

$$\begin{aligned} x_1'(\eta_1) =&\lambda\Bigg[-\frac{a_{11}(\eta_1)}{e^{x_1(\eta_1)}} + \frac{b_{11}(\eta_1)}{e^{x_1(\eta_1)+e^{x_1(\eta_1)}}} + \frac{a_{12}(\eta_1)}{e^{x_1(\eta_1)}} - \frac{b_{12}(\eta_1)}{e^{x_1(\eta_1)+e^{x_2(\eta_1)}}} \\ &+\sum_{j=1}^{l} c_{1j}(\eta_1)\frac{e^{x_1(\eta_1-\tau_{1j}(\eta_1))}}{e^{x_1(\eta_1)+\gamma_{1j}(\eta_1)e^{x_1(\eta_1-\tau_{1j}(\eta_1))}}}\Bigg] \\ =&0. \end{aligned} \tag{2.12}$$

In view of the fact that $\sup\limits_{u\geq 0} ue^{-u} = \frac{1}{e}$, one can get

$$\begin{aligned}
a_{11}^{-} - a_{12}^{+} \leq & a_{11}(\eta_1) - a_{12}(\eta_1) + \frac{b_{12}(\eta_1)}{e^{e^{x_2(\eta_1)}}} \\
= & \frac{b_{11}(\eta_1)}{e^{e^{x_1(\eta_1)}}} + \sum_{j=1}^{l} c_{1j}(\eta_1) \frac{\gamma_{1j}(\eta_1) e^{x_1(\eta_1 - \tau_{1j}(\eta_1))} e^{-\gamma_{1j}(\eta_1) e^{x_1(\eta_1 - \tau_{1j}(\eta_1))}}}{\gamma_{1j}(\eta_1)} \\
\leq & \frac{b_{11}^{+}}{e^{e^{x_1(\eta_1)}}} + \sum_{j=1}^{l} \frac{c_{1j}^{+}}{\gamma_{1j}^{-} e},
\end{aligned} \tag{2.13}$$

it follows from (2.1) and (2.13) that

$$x_1(\eta_1) \leq H_{12}, \tag{2.14}$$

where H_{12} is a fixed constant satisfying

$$a_{11}^{-} - a_{12}^{+} \geq \frac{b_{11}^{+}}{e^{e^{H_{12}}}} + \sum_{j=1}^{l} \frac{c_{1j}^{+}}{\gamma_{1j}^{-} e}.$$

In the same way, we can obtain

$$x_2(\xi_2) \geq \ln \left(\ln \frac{b_{22}^{-}}{a_{22}^{+} + b_{21}^{+} - a_{21}^{-}} \right) := H_{21}, \tag{2.15}$$

and

$$x_2(\eta_2) \leq H_{22}, \tag{2.16}$$

where H_{22} is a fixed constant satisfying

$$a_{22}^{-} - a_{21}^{+} \geq \frac{b_{22}^{+}}{e^{e^{H_{22}}}} + \sum_{j=1}^{l} \frac{c_{2j}^{+}}{\gamma_{2j}^{-} e}.$$

Let $H > \max\{|H_{11}|, |H_{21}|, |H_{12}|, |H_{22}|\}$ be a fix constant and define $\Omega = \{x \in X : ||x|| < H\}$. Then (2.11), (2.14), (2.15) and (2.16) imply that there is no $\lambda \in (0,1)$ and $x \in \partial\Omega$ such that $Lx = \lambda Nx$. If $x(t) = (x_1(t), x_2(t))^T \in \partial\Omega \cap KerL$, then $x(t)$ is a constant vector in R^2, and there exists some $i \in \{1,2\}$, such that $|x_i| = H$. Assume $|x_1| = H$, so that $x_1 = \pm H$. Then, we claim

$$(QN(x))_1 > 0 \text{ for } x_1 = -H, \quad \text{and} \quad (QN(x))_1 < 0 \text{ for } x_1 = H. \tag{2.17}$$

If $(QN(x))_1 \leq 0$ for $x_1 = -H$, it follows from (2.1) and (2.5) that

$$\int_0^{\omega} \Delta_1(x,t)dt \leq 0, \text{ for } x_1 = -H.$$

Hence,

$$\begin{aligned}(a_{11}^{+}+b_{12}^{+}-a_{12}^{-})e^{H} \geq & \frac{1}{\omega}\int_0^{\omega}[\frac{a_{11}^{+}}{e^{-H}}+\frac{b_{12}^{+}}{e^{-H+e^{x_2(t)}}}-\frac{a_{12}^{-}}{e^{-H}}]dt \\ \geq & \frac{1}{\omega}\int_0^{\omega}[\frac{a_{11}(t)}{e^{-H}}+\frac{b_{12}(t)}{e^{-H+e^{x_2(t)}}}-\frac{a_{12}(t)}{e^{-H}}]dt \\ \geq & \frac{1}{\omega}\int_0^{\omega}[\frac{b_{11}(t)}{e^{-H+e^{-H}}}+\sum_{j=1}^{l}\frac{c_{1j}(t)}{e^{\gamma_{1j}(t)e^{-H}}}]dt \\ > & \frac{b_{11}^{-}}{e^{-H+e^{-H}}},\end{aligned}$$

which yields

$$-H > \ln\left(\ln\frac{b_{11}^{-}}{a_{11}^{+}+b_{12}^{+}-a_{12}^{-}}\right) = H_{11}.$$

This is contradiction and implies that $(QN(x))_1 > 0$ for $x_1 = -H$.

If $(QN(x))_1 \geq 0$ for $x_1 = H$, from (2.1), (2.5) and $\sup_{u\geq 0} ue^{-u} = \frac{1}{e}$, we get

$$\int_0^{\omega}\Delta_1(x,t)dt \geq 0,\ \text{ for } x_1 = H,$$

and

$$\begin{aligned}(a_{11}^{-}-a_{12}^{+})e^{-H} < & \frac{1}{\omega}\int_0^{\omega}\left[\frac{a_{11}(t)}{e^{H}}-\frac{a_{12}(t)}{e^{H}}+\frac{b_{12}(t)}{e^{H+e^{x_2(t)}}}\right]dt \\ = & \frac{1}{\omega}\int_0^{\omega}\left[\frac{b_{11}(t)}{e^{H+e^{H}}}+\sum_{j=1}^{l}\frac{c_{1j}(t)\gamma_{1j}(t)e^{H}e^{-\gamma_{1j}(t)e^{H}}}{\gamma_{1j}(t)e^{H}}\right]dt \\ \leq & \frac{b_{11}^{+}}{e^{H+e^{H}}}+\sum_{j=1}^{l}\frac{c_{1j}^{+}}{\gamma_{1j}^{-}e^{H}e}.\end{aligned}$$

Consequently,

$$H < \ln\left(\ln\frac{b_{11}^{+}}{a_{11}^{-}-a_{12}^{+}-\sum_{j=1}^{l}\frac{c_{1j}^{+}}{\gamma_{1j}^{-}e}}\right) = H_{12},$$

a contradiction to the choice of H. Thus, $(QN(x))_1 < 0$ for $x_1 = H$.

Similarly, if $|x_2| = H$, we obtain

$$(QN(x))_2 > 0 \text{ for } x_2 = -H, \quad \text{and} \quad (QN(x))_2 < 0 \text{ for } x_2 = H. \tag{2.18}$$

Furthermore, let $0 \leq \mu \leq 1$ and define a continuous function $H(x,\mu)$ by setting

$$H(x,\mu) = -(1-\mu)x + \mu QNx.$$

It follows from (2.17) and (2.18) that $H(x,\mu) \neq (0,0)^T$ for all $x \in \partial\Omega \cap kerL$. Hence, using the homotopy invariance theorem, we obtain

$$\deg\{QN, \Omega \cap kerL, (0,0)^T\} = \deg\{-x, \Omega \cap kerL, (0,0)^T\} \neq 0.$$

It then follows from the continuation theorem that $Lx = Nx$ has a solution

$$x^*(t) = (x_1^*(t), x_2^*(t))^T \in DomL \bigcap \overline{\Omega},$$

which is an ω-periodic solution to system (2.3). Therefore $N^*(t) = (N_1^*(t), N_2^*(t))^T = (e^{x_1^*(t)}, e^{x_2^*(t)})^T$ is a positive ω-periodic solution of (1.2) and the proof is complete.

3 Example and Remark

In this section, we give an example to illustrate the results obtained in the previous section.

Example 3.1. Consider the following Nicholson-type delay system with nonlinear density-dependent mortality terms:

$$\begin{cases} N_1'(t) = -(4+\sin t) + (8 + |\cos t|)e^{-N_1(t)} + (1+\cos t) - (1+|\sin t|)e^{-N_2(t)} \\ \qquad + e^{4\pi}(1+\frac{\cos t}{4})N_1(t-|2+\cos t|)e^{-e^{4\pi+|\sin t|}N_1(t-|2+\cos t|)} \\ \qquad + e^{4\pi}(1+\frac{\sin t}{4})N_1(t-|2+\sin t|)e^{-e^{4\pi+|\cos t|}N_1(t-|2+\sin t|)}, \\ N_2'(t) = -(6+\cos t) + (10 + |\sin t|)e^{-N_2(t)} + (2+\sin t) - (1-|\cos t|)e^{-N_1(t)} \\ \qquad + e^{4\pi}(1+\frac{\sin t}{4})N_2(t-|2+\sin t|)e^{-e^{4\pi+|\cos t|}N_2(t-|2+\sin t|)} \\ \qquad + e^{4\pi}(1+\frac{\cos t}{4})N_2(t-|2+\cos t|)e^{-e^{4\pi+|\sin t|}N_2(t-|2+\cos t|)}, \end{cases} \tag{3.1}$$

Obviously,

$$a_{11}^- = 3, a_{11}^+ = 5, b_{11}^- = 8, b_{11}^+ = 9, a_{12}^- = 0, a_{12}^+ = 2, b_{12}^+ = 2,$$

$$a_{22}^- = 5, a_{22}^+ = 7, b_{22}^- = 10, b_{22}^+ = 11, a_{21}^- = 1, a_{22}^+ = 3, b_{21}^+ = 2,$$

$$c_{ij}^+ = \frac{5}{4}e^{4\pi}, \qquad \gamma_{ij}^- = e^{4\pi} \quad (i,j = 1,2),$$

then

$$\frac{b_{11}^-}{a_{11}^+ + b_{12}^+ - a_{12}^-} = \frac{8}{7} > 1, \qquad a_{11}^- > a_{12}^+ + \sum_{j=1}^{l} \frac{c_{1j}^+}{\gamma_{1j}^- e},$$

$$\frac{b_{22}^-}{a_{22}^+ + b_{21}^+ - a_{21}^-} = \frac{5}{4} > 1, \qquad a_{22}^- > a_{21}^+ + \sum_{j=1}^{l} \frac{c_{2j}^+}{\gamma_{2j}^- e}.$$

Consequently, all the conditions in Theorem 2.1 hold. Therefore, system 3.1 has at least one 2π-periodic solution with strictly positive components.

Remark 3.1. *To our knowledge, few authors have studied the problems of positive periodic solution of Nicholson's blowflies delayed systems with nonlinear density-dependent mortality terms. It is clear that the results in [3,4,7,14,17-18] and the references therein cannot be applicable to system (3.1) to prove the existence of positive periodic solution. Moreover, one can find that the main results of [16] is restricted to consider the Nicholson's blowflies delayed systems with non-linear density-dependent mortality terms* $\frac{a_{ij}(t)N}{b_{ij}(t)+N}$ *and give no opinions about* $a_{ij}(t) - b_{ij}(t)e^{-N}$*. This implies that the results of the present paper are new and complement previously known results.*

Acknowledgement

The author would like to express the sincere appreciation to the reviewers for their helpful comments in improving the presentation and quality of the paper.

References

[1] W.S.C Gurney, S.P. Blythe, R.M. Nisbet, Nicholson's blowflies revisited, Nature, 287 (1980) 17-21.

[2] A.J. Nicholson, An outline of the dynamics of animal populations, Aust. J. Zool., 2 (1954) 9-65.

[3] Y. Chen, Periodic solutions of delayed periodic Nicholson's blowflies models, Can. Appl. Math. Q., 11 (2003) 23-28.

[4] J. Li, C. Du, Existence of positive periodic solutions for a generalized Nicholson's blowflies model, J. Comput. Appl. Math. 221 (2008) 226-233.

[5] W. Chen, B.Liu, Positive almost periodic solution for a class of Nicholson's blowflies model with multiple time-varying delays, J. Comput. Appl. Math. 235 (2011) 2090-2097.

[6] B. Liu, Global stability of a class of Nicholsons blowflies model with patch structure and multiple time-varying delays, Nonlinear Anal. Real World Appl. 11 (4) (2010) 2557-2562.

[7] B. Liu, The existence and uniqueness of positive periodic solutions of Nicholson-type delay systems, Nonlinear Anal. Real World Appl. 12 (2011) 3145-3151.

[8] W. Zhao, C. Zhu, H. Zhu, On positive periodic solution for the delay Nicholson's blowflies model with a harvesting term, Appl. Math. Modelling 36 (2012) 3335-3340.

[9] T. Yi, X. Zou, Global attractivity of the diffusive Nicholson's blowflies equation with Neumann boundary condition: A non-monotone case, J. Differential Equations 245 (2008) 3376-3388.

[10] Y. Yang, R. Zhang, C.Jin and J.Yin, Existence of time periodic solutions for the Nicholson's blowflies model with Newtonian diffusion, Math. Method. Appl. Sci. 33 (2010) 922-934.

[11] X. Liu, J. Meng, The positive almost periodic solution for Nicholson-type delay systems with linear harvesting terms, Appl. Math. Modelling 36 (2012) 3289-3298.

[12] L. Berezansky, L. Idels, L. Troib, Global dynamics of Nicholson-type delay systems with applications, Nonlinear Anal. Real World Appl. 12 (1) (2011) 436-445.

[13] W. Wang, W. Chen, L. Wang, Existence and exponential stability of positive almost periodic solution for Nicholson-type delay systems, Nonlinear Anal. Real World Appl. 12 (2011) 1938-1949.

[14] B. Liu, The existence and uniqueness of positive periodic solutions of Nicholson-type delay systems, Nonlinear Anal. Real World Appl. 12 (2011) 3145-3151.

[15] L. Berezansky, E. Braverman, L. Idels, Nicholson's Blowflies Differential Equations Revisited: Main Results and Open Problems, Appl. Math. Modelling 34 (2010) 1405-1417.

[16] W. Chen, L. Wang, Positive Periodic Solutions of Nicholson-type Delay Systems with Nonlinear Density-dependent Mortality Terms, Hindawi Publishing Corporation, Abstr. Appl. Anal. (2012) ID 843178, doi:10.1155/2012/843178.

[17] X. Hou, L. Duan, New Results on Periodic Solutions of Delayed Nicholson's Blowflies Models, Electron. J. Qual. Theory Differ. Equ. 24 (2012) 1-11.

[18] X. Hou, L. Duan, Z. Huang, Permanence and periodic solutions for a class of delay Nicholson's blowflies models, Appl. Math. Modelling (2012), doi: 10.1016/j.apm.2012.04.018.

[19] B. Liu, S. Gong, Permanence for Nicholson-type delay systems with nonlinear density-dependent mortality terms, Nonlinear Anal. Real World Appl. 12 (2011) 1931-1937.

[20] R.E. Gaines, J. Mawhin, Coincide degree and nonlinear differential equations, Lecture Notes in Math, vol. 568, Spring, Berlin, 1977.

(Received June 1, 2012)